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Pinning of cracks in two-dimensional disordered media

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Abstract. We study the statistics of crack pinning in two dimensions by experiments and simulations of directed polymers in random media. Mode I tensile tests on paper samples show a delocalization phenomenon as the notch length is varied if the fraction of cracks pinned to the notch is monitored. This is compared with the behavior of directed polymers in the presence of both an energetically favorable localized pinning center and bulk disorder. An analysis of the crack interface roughness indicates self-affine behavior with a roughness exponent ζ in the proximity of the minimum energy surface value 2/3.

PACS. 62.20.Mk Fatigue, brittleness, fracture, and cracks – 62.20.Fe Deformation and plasticity (including yield, ductility, and superplasticity) – 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 81.40.Np Fatigue, corrosion fatigue, embrittlement, cracking, fracture and failure

1 Introduction

Recent experiments have shown that novel aspects of fracture phenomena, in particular in disordered media, transpire if one uses the language of statistical mechanics [1,2]. Two main new ideas have been encountered. The first issue has been the growth and formation of cracks and in general questions related to extensive properties like the tensile strength. Secondly, the *a posteriori* properties of fracture surfaces have attracted much interest [1,3]. The geometric characterization of fractures has resulted in a multitude of evidence for non-trivial self-affine scaling though the details vary greatly. The properties of crack surfaces could be assumed to depend on the dynamics of crack formation. Of particular interest is the connection to standard depinning phenomenology, which might be relevant in the case in which the crack is formed slowly with elastic energy release competing with microstructural and other randomness [4]. This establishes a connection between the physics of driven elastic lines in random media and the propagation of cracks and helps to introduce to fracture the standard plethora of scaling exponents [5]. In this way many fracture mechanics problems can just be considered in standard terms of driven interfaces, complicated due to the long-range nature of the elastic forces. In a disordered material the competition of elasticity with the inhomogeneities in the material leads to generic scaling laws. Close to a depinning transition one would observe a power-law dependency of the crack velocity on the drive force, and possibly self-affine rough cracks whose roughness is in three dimensions characterized by a set of roughness exponents – in-plane, out-of-plane – with possibly different values in the two independent directions.

However, such a clear picture is according to current understanding not in agreement with how disorder and dimensionality affect fracture processes. For three dimensional fracture, it has been demonstrated that crack interfaces obey at long enough lengthscales self-affine scaling, with a roughness (Hurst) exponent ζ close to 0.8 [1]. For small scales the exponent seems to be around 0.5. These values are difficult to explain by the phenomenology of driven lines in random media. In two dimensions the situation is for reasons not yet understood different, in that both experiments on slow, tensile failure of paper sheets and on a quasi-2D system lead to $\zeta \simeq 0.6-0.7$ [6,7].

The implications of having in 2D such a super-diffusive roughness exponent (it is larger than the random walk value of 1/2) are interesting since the actual quotes for ζ are close to the directed polymer (DP) in a random medium or minimum energy surface exponent, $\zeta_{\rm dp} = 2/3$ [8]. It is unclear how exactly a pinning cluster or a minimum energy surface (corresponding to a directed polymer) maps to a fracture surface but numerical evidence exists that 2D slow crack growth is in the DP universality class [9,10]. In the case of perfect (scalar) plasticity the corresponding random fuse network (RFN) model possesses a blocking property. In a ductile medium the global yield point is reached once an intersecting surface is formed that blocks any further increases in stress (current in a scalar approximation). This surface is exactly equivalent to a random bond Ising domain wall, which is in the DP universality class. Recent computer simulations by Räisänen *et al.* indicate this to be true also for brittle failure, and thus perhaps also for the whole continuum

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for scalar models ranging from brittle to perfectly plastic behavior [9,10].

In this paper we extend the analogy of directed polymers and fracture problems by studying the failure of systems with prepared defects or notches. Consider a random bond Ising domain wall (DW), *e.g.* in a $L \times L$-system with antiperiodic boundary conditions (spins fixed up/down at opposite ends). Recall that this is equivalent to a perfectly ductile material. The Hamiltonian of the interface is given by

$$H = -\sum_{nn'} J_{nn'} \sigma_n \sigma'_n, \tag{1}$$

where the sum is over nearest-neighbor spin pairs and the $J_{nn'}$ are positive semidefinite random variables with a sensible distribution (finite first and second moments). The interface energy is minimized at zero temperature by the DW configuration that chooses the global optimum by utilizing small coupling constants and by not expending energy by increasing the total DW length too much. What happens in this case if one sets the bonds J_{xy} to zero at one end ($x = L/2$, $y \leq a$)?

This pinning defect should correspond in fracture mechanics language to a notch. The first question to ask is: when is the strength of the defect strong enough to localize the interface/crack so that it passes through the defect given a fixed system size L? This question will be studied in the following experimentally by tensile tests on notched 2D paper samples with varying notch lengths. These are compared to simulations of directed polymers in a set-up as above with bulk disorder and a defect. One should of course keep in mind that such a scalar and perfectly plastic model lacks some features of real materials. The 'order parameter' used in this work is the probability for the interface to pass through the notch, f. Out of the simulations and the experiments seems to emerge a 'delocalization transition' as the scaling parameter a/L is varied. Recall that the defect is finite and thus a sensible limit for finite size scaling consists of $a/L \to$ const. This is since the problem of a DP with a finite defect is related to the question of DP's interacting with extended defects. It is known that in 2D an arbitrarily weak bulk line defect is relevant and localizes the interface (see [11] and also [8]). What corresponds to this in our problem is the above limit of a constant pin length/system size ratio.

The rest of the paper is structured so that Section 2 discusses the problem and the scaling of the fraction of pinned cracks in the light of the directed polymer analogy and computer simulations. Section 3 presents the experimental details and the data for both crack pinning and the roughness of final crack interfaces. Section 4 finishes the paper with conclusions.

2 Directed polymers and localized defects

Consider the Ising domain wall problem starting from equation (1). Define the function $f(a/L)$ as the fraction of interfaces going through the defect. Figure 1 shows the

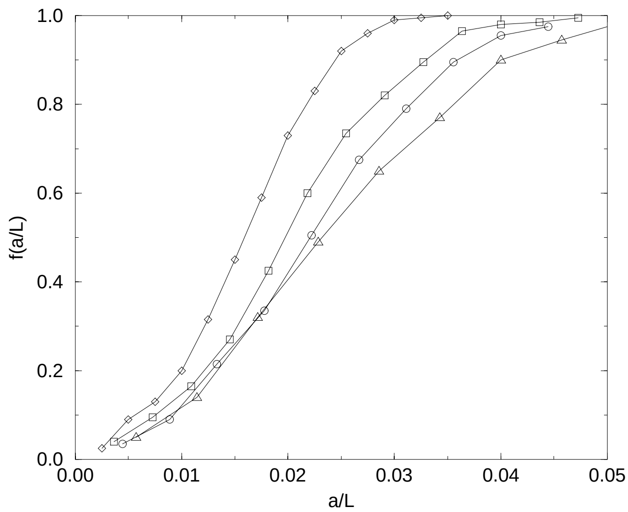

Fig. 1. f *vs.* L from simulations as a function of system size and notch/defect length a. The system size L increases from right-to-left, L = 100, 200, 300, 400 (square geometry). Typically 200 samples per datapoint in the numerics.

behavior of f with system size. We use in the numerics a combinatorial optimization technique, which allows a convenient way of finding minimum energy surfaces with boundary conditions that allow for free starting and end points for the interface [12]. The systems have square geometry and the bulk disorder used is given by a constant bond probability distribution $[1 - \Delta J \leq J \leq 1 + \Delta J]$. With increasing L, f approaches a linear function of a except for close to zero and unity, respectively. The slope is proportional to the system size L.

In the limits $f \to 1$ and $f \to 0$ the scaling behavior is less trivial than in the linear regime seen in Figure 1. For the linear part, one can however use simple scaling arguments. For an arbitrary starting point (x, y) for the DP the interface energy can be written in the form $g_{x,y}(\delta E)(L - x)$, $x \ll L$, where of the arguments the x-coordinate is in the direction along the notch and the y-coordinate is the perpendicular one. g measures the mean effect of the actual initial condition on the DP energy while δE is the average DP energy per unit length. If one now adds a zero-energy defect of length a to the original system, $f(a) \simeq 0.5$ corresponds to a crossover value a_c for the crack length. This can be found by considering the two candidate energies for the minimum path: $\Delta E_1 = g_{1,y}L$ and $\Delta E_2 = (L - a_c)g_{a,L/2}$. $g_{a,L/2}$ takes into account that the energy of the path constrained to pass through the notch should be larger than that of the unconstrained one. The crack is as likely to be unpinned as pinned when the energy lost by using the minimum path of the pure system is typically as large as that of the minimum path using the defect, *i.e.* $\Delta E_1 = \Delta E_2$. The competition of the bulk disorder and the defect with varying defect strength can be compared by expanding $g \sim 1 - \Delta J g'_{x,y}$ where g' is to the first order constant in

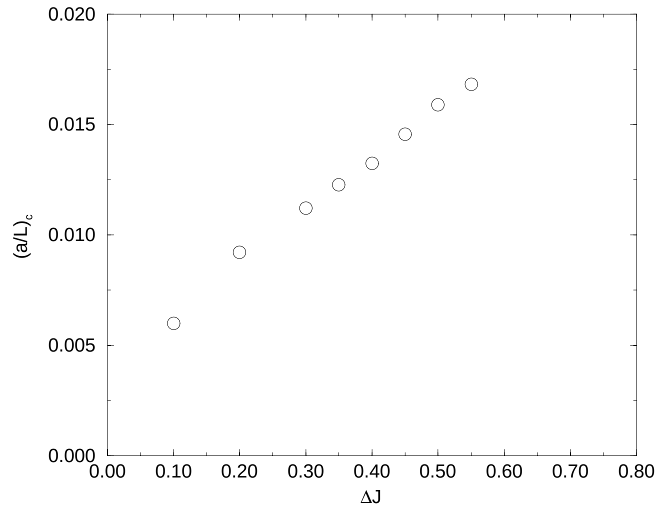

Fig. 2. The critical defect size a_{c} from simulations with $L = 200$ as a function of disorder strength $\Delta J = 0.5$.

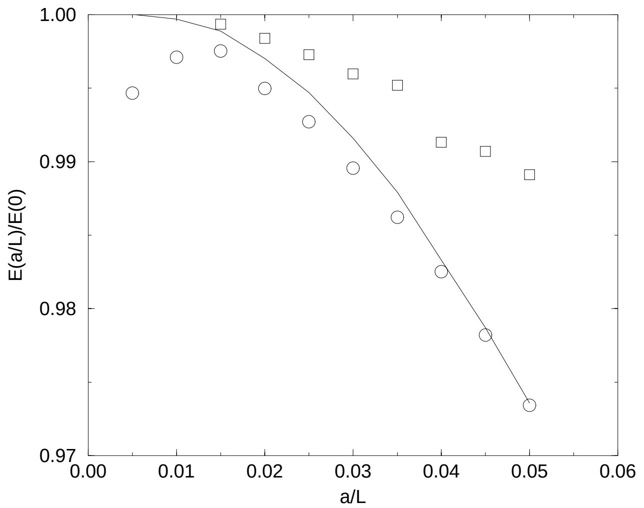

Fig. 3. Directed polymer energy $E(a)$ *vs.* defect size a normalized with the energy of the system with no defect $E(0)$. The circles denote the average computed over systems in which the defect is included in the surface, and the squares those in which it is avoided. The line denotes the total average $E(a)$. $\Delta J = 0.5$, thus $a_{\mathrm{c}}/L = 0.017$, $L = 200$.

x and determined solely by y. Trivial algebra gives

$$a_{\mathrm{c}} = \frac{\Delta J(g'_y - g'_{L/2})}{1 - \Delta J g'_{L/2}} L. \qquad (2)$$

For ΔJ small the cross-over lengthscale is thus linear in ΔJ; this is compared with simulations in Figure 2. The tails of f close to pinning and complete depinning are difficult to argue about since one needs to take into account that the minimum energy of 'through the notch' cracks in a sample will both depend on the actual minimum energy without the notch E_0 as well as the interface configuration corresponding to that. Since the tails of DP energy distribution functions are (within the accuracy we are concerned with) Gaussian a rough estimate would be about the limit $f \to 1$ that $1 - f$ would be Gaussian as well. For $f \to 0$ one can not assume that the global minimum and the one emanating from the defect are well-separated in space.

Figure 3 shows the average interface energy in three different ways: *vs.* a, and, separately, the averages for pinned and unpinned samples. They are all normalized with the average interface energy without a defect. There are two things to note: first, the samples in which the interface utilized the defect are on average weaker (energy per unit length smaller) than those with the same defect size/strength but without the interface passing through the defect. Second, the average energy for the unpinned cases decreases with a/L. The latter fact makes possible a qualitative explanation of the first observation. There is a certain fraction of samples in which any interface path that uses the defect is energetically very unfavourable ($g_{a,L/2}$ large). In such cases the actual global minimum is more likely to win the competition. The energy – given the added constraint – of such systems is larger on the average, but of course decreases as well with increasing a/L.

3 Experimental results

3.1 Crack pinning

Ordinary paper is inhomogeneous, which can be affirmed with the naked eye. This unevenness arises from both simple variations in the local mass (flocs, basis weight fluctuations) and in the local mechanical properties as failure strain or elastic modulus. The role of these fluctuations in the strength properties is an open question. Experimental observations indicate that the local strength is not directly proportional to the local basis weight (mass per unit area). As in any engineering material the disorder present gives rise to complications. At the simplest level weak-link statistics plays a role in determining the tensile strength of samples: the larger the sample, the lower its strength due to variations of the local strength locally. The inhomogeneity at the structural level could become visible in the variation/probability distribution of the tensile strength. Broadly speaking the more inhomogeneous the local structure, the wider the spread in the tensile strengths would be.

The issue of the role of the defects/unevenness of structure has previously been noted in paper physics literature [13,14]. One can attempt to account for the structural aspects by adding a material-specific correction to the effective linear-elastic fracture mechanics crack size, δa_{s} to come up with a modified Griffith's equation [14]. A side product of the analysis is that one can define an effective disorder-induced crack size. *E.g.*, one concept is to use the size of a notch that will have 50% of the cracks passing through it a_{c}, in our language such that $f(a_{\mathrm{c}}) \simeq 0.5$.

The experiments are done with two paper grades: newsprint and copy paper of 45 and 70 $\mathrm{g/m^2}$ basis

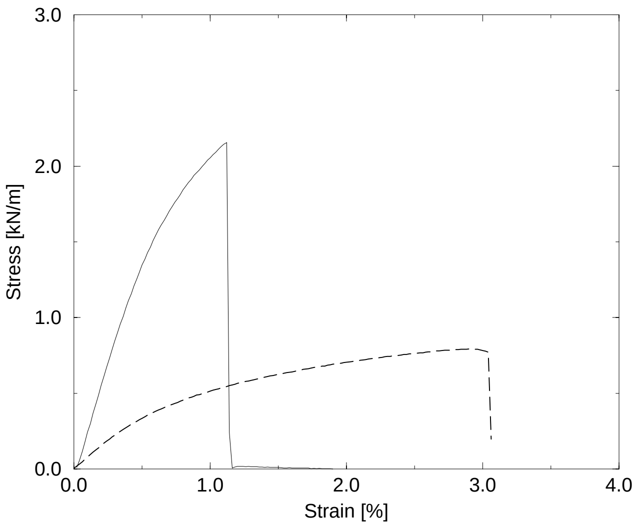

Fig. 4. Typical stress-strain curves of the paper grades used. The solid curve: newsprint in the machine-direction (MD), the dashed curve: newsprint in the cross-direction (CD), see the text.

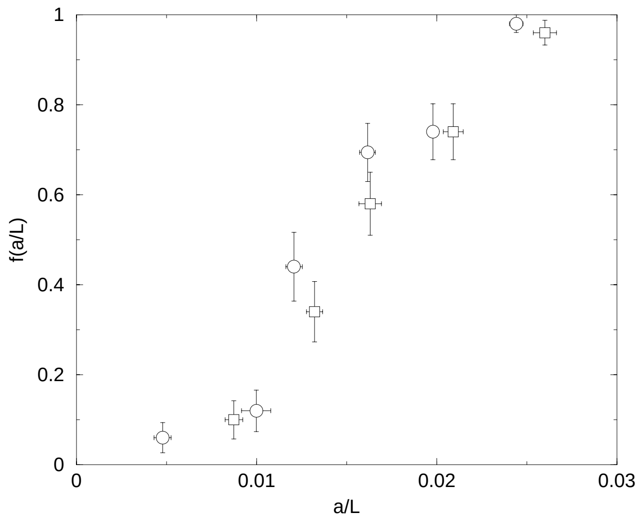

Fig. 5. The experimental f, pinning fraction *vs.* the notch length normalized with system size a/L. The disorder scale a_c/L is about 0.013. The error bars denote the experimental scattering in the notch length and in the f. The data presented is for two separate datasets for copy paper in the CD (more ductile) direction (circles: paper stored in room conditions, squares: paper stored in constant temperature and relative humidity to avoid aging).

weight, respectively. Figure 4 shows the typical stress-strain curves of unnotched samples of newsprint and demonstrates that the paper grades vary in their relative ductility. One issue to note is that both variants are of industrial origin and thus display an anisotropy (in the so-called Machine and Cross Directions, MD/CD) in elastic modulus, strength and strain to failure etc. The grades considered here are the most ductile if strained in the CD.

Notches of varying length were cut on the edges of 100×100 mm test samples. The edge cut lengths were measured afterwards by using a microscope to a 0.05 mm accuracy. The effective accuracy of the notch length is 0.2 mm. The tensile tests were carried out with a standard Alwetron tensile testing machine at the Paper Technology laboratory of Helsinki University of Technology. The samples – 50 for each notch length – were stored at constant temperature and humidity to avoid viscoelastic changes which might change the effective ductility of the material over the test period. Each sample was subjected to a constant strain rate of 1% per minute tensile test.

Figure 5 shows $f(a/L)$ for one of the paper grades. The slope of f in the linear regime turns out to be larger for the ductile copy paper. This might be due to less inherent disorder. From the data one can estimate the defect size a at which f becomes effectively zero. This turns out to be of the order of 0.7–0.8 mm for all cases except for MD copy paper for which the a may be smaller, close to zero. The disorder-induced crack size a_c is of the order of 1.1 to 1.3 mm depending on the paper grade (copy paper or newsprint, respectively) but not depending on the orientation. Thus (see Fig. 4) the macroscopic ductility of the samples in the direction of strain seems to play no great role in the magnitude of a_c.

The strength of the samples was found to decrease roughly linearly as a function of a/L. Despite the limited statistics one can gain extra information of the two populations (pinned/depinned) by comparing the relative strengths when $f \sim 0.5$. Analogous to the simulations the strength of the pinned samples is lower than of the depinned ones (at a 95% confidence level). It must be emphazised that the comparison would be well-defined only for a very ductile material; for brittle ones the failure stress may or may not have any direct relation to the average interface energy.

3.2 Fractureline analysis

To explore further the analogy between minimum energy surfaces and the cracks we analyzed the crack roughness of a subset of the tensile test samples. The CD copy paper data was used with the mean notch length value of 27 mm. Five (samples 1–5) of the ten fracturelines did not include the initial notch and the other five (samples 6–10) included the notch, *i.e.* the notch length was chosen so as to imply $f \sim 0.5$, $a \simeq a_c$.

The strips were scanned with 600 dpi tablescanner. The power of reflected light was digitized using 256 gray levels. Unfortunately the grayscale range was set so that images actually contain only 62–64 distinct gray levels. The characteristic image size is 2350×500 pixels.

The fracturelines were detected by thresholding the image. The thresholding graylevel varied between samples. The threshold value was determinated by fitting one Gaussian distribution to 0-128-part of graylevel histogram and another Gaussian distribution to 128-256-part. The threshold value is the graylevel that has the equal probability to belong to both of the distributions.

Table 1. Effective roughness exponent ζ values from crack surface analysis of the experimental data with $a \sim a_c$, $f \sim 0.5$ (see text). measurements.

Method	Excluding notch (1–5)		Including notch (6–10)	
	Mean	Var.	Mean	Var.
Return prob.	0.84	0.03	0.80	0.02
$z_{\max}$	0.70	0.006	0.66	0.009
Width	0.62	0.01	0.53	0.003
Width trend removed	0.57	0.02	0.51	0.006

The fracturelines were solid-on-solid -conditioned and steps greater than 1 mm were removed. Since the sample dimensions were small and the test method obviously preferred one crack propagation direction the global linear trend was not removed. To demonstrate the effect of trend removal the standard deviation method was applied to both the original and detrended fractureline. We used several test methods for the crack roughness analysis: local width, maximum deviation ('$z_{\max}$'), first return probability and Fourier-analysis. These all should result in the same roughness exponent ζ if the crack surfaces are truly self-affine.

The fracturelines were analyzed using the three methods. All these show that the fractureline becomes slightly rougher if the fractureline does not include the initial notch (Tab. 1); note that for DP's with a linear defect this would be trivially the case. It seems that the variation of roughness exponent is higher in those samples that included the initial notch.

The exponent values obtained are in spite of their large relative variation in rough agreement with two assumptions: 1) the crack surfaces are self-affine and 2) that the roughness exponent should be of the order of 2/3, as for directed polymers.

4 Conclusions

In this paper we have compared the behavior in slow fracture of disordered two-dimensional media and the physics of minimum energy surfaces. To this end, we have defined an order parameter f describing the mean fraction of samples in which the crack or the directed polymer passes through the crack. From the fracture mechanics viewpoint, $f(a)$ describes the 'intrinsic' disorder in the material in a semi-quantitative way whereas for minimum energy surfaces it can be related to the behavior in the presence of a defect of finite extent.

Both from the experiments and the DP simulations a similar picture emerges: there is a regime in which $f(a)$ is roughly linear in a, and one may extract a 'typical defect size' from this as the notch/defect size a_c at which f extrapolates to zero. For the particular case studied here, ordinary paper of industrial manufacture this lengthscale is of the order of 0.5 mm, and in other words non-zero. As pointed out in reference [14], a length scale like a_c can perhaps be considered to related to the 'plastic zone correction' of linear elastic fracture mechanics of materials which exhibit plasticity in the fracture process zone. This means simply that for such materials the expression for the critical stress $\sigma_c \sim 1/\sqrt{a + \delta a}$ where δa is the lengthscale of microscopic plasticity, the implication in our case being that $\delta a \sim a_c$. Thus the larger the a_c, the tougher would be the material. Note also the scenario presented in Figure 3: the tendency to not pin to the defect is related to the lack of any 'easy' paths that could be accessed. The roughness analysis of the cracks for a case in which $f \sim 0.5$ results in exponents that are not in obvious disagreement with a minimum energy surface-like behavior (ζ being of the order of 0.6...0.7). Finally we mention some future prospects: the idea of the paper can be naturally extended to three dimensions, where a 2D crack plane would interact with a 2D notch. It would be also interesting to investigate systematically the 'scaling limit' of a constant a/L-ratio.

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